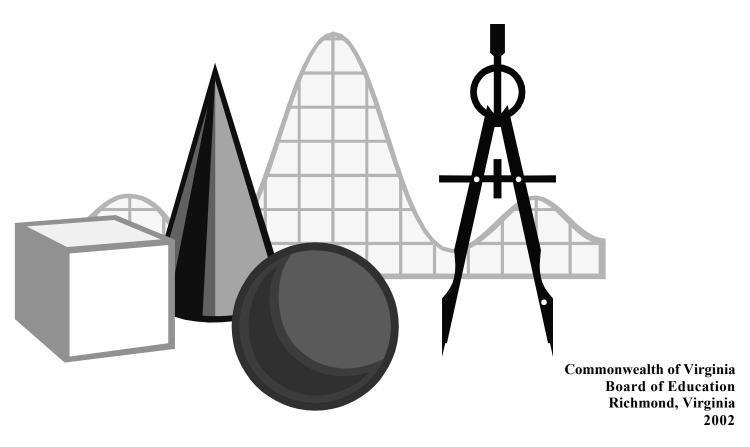
# MATHEMATICS STANDARDS OF LEARNING SAMPLE SCOPE AND SEQUENCE

Algebra II



Copyright © 2002

by the

Virginia Department of Education P.O. Box 2120 Richmond, Virginia 23218-2120 www.pen.k12.va.us

All rights reserved. Reproduction of materials contained herein for instructional purposes in Virginia classrooms is permitted.

#### **Superintendent of Public Instruction**

Jo Lynne DeMary

#### **Deputy Superintendent**

M. Kenneth Magill

#### **Assistant Superintendent for Instruction**

Patricia I. Wright

#### **Office of Secondary Instructional Services**

Linda M. Wallinger, Director Deborah Kiger Lyman, Mathematics Specialist

#### NOTICE TO THE READER

The Virginia Department of Education does not unlawfully discriminate on the basis of sex, race, color, religion, handicapping conditions, or national origin in employment or in its educational programs and activities.

The Mathematics Standards of Learning Sample Scope and Sequence and the Mathematics Standards of Learning Teacher Resource Guide can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at <a href="http://www.pen.k12.va.us">http://www.pen.k12.va.us</a>.

#### **Preface**

As an additional resource to help school divisions develop curricula aligned to the 2001 Mathematics Standards of Learning, the Virginia Department of Education has developed sample scope and sequence documents in kindergarten through grade eight and in core high school courses. These sample documents provide guidance on how the essential knowledge and skills that are identified in the Standards of Learning and the Standards of Learning Curriculum Framework may be introduced to students in a logical, sequential, and meaningful manner.

These sample scope and sequence documents are intended to serve as general guides to help teachers and curriculum developers align their curricula and instruction to support the Standards of Learning. Each sample document is organized around specific topics to help teachers present information in an organized, articulated manner. Also included are correlations to the Standards of Learning for that curricular area for a particular grade level or course, as well as ideas for classroom assessments and teaching resources.

The sample scope and sequence documents are not intended to prescribe how curriculum should be developed or how instruction should be delivered. Instead, they provide examples showing how teachers and school divisions might present to students in a logical and effective manner information that has been aligned with the Standards of Learning. School divisions that need assistance in developing curricula aligned with the Standards of Learning are encouraged to consider the sample scope and sequence guides. Teachers who use the documents should correlate the content identified in the guides with available instructional resources and develop lesson plans to support instruction.

Copies of the sample scope and sequence guides are available at <a href="http://www.pen.k12.va.us">http://www.pen.k12.va.us</a> in both PDF and Microsoft Word formats. These materials are copyrighted, and all rights are reserved. Reproduction of these materials for instructional purposes in Virginia classrooms is permitted.

#### Introduction

Mathematics content develops sequentially in concert with a set of processes that are common to different bodies of mathematics knowledge. The content of the Mathematics Standards of Learning supports four process standards as goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, and making mathematical connections. These goals provide a context within which to develop the knowledge and skills identified in the standards.

Students should be helped to make connections and to build relationships between algebra, arithmetic, geometry, discrete mathematics, and probability and statistics. Connections should be made to other subject areas and fields of endeavor through applications. Using manipulatives, graphing calculators, and computer applications to develop concepts should help students develop and attach meaning to abstract ideas. Throughout the study of mathematics, students should be encouraged to talk about mathematics, to use the language and symbols of mathematics, to communicate, to discuss problems and problem solving, and to develop their competence and their confidence in themselves as mathematics students.

This document is intended as a general guide to help teachers and schools frame a curriculum that incorporates the fundamentals of secondary mathematics courses and to provide a correlation of those fundamentals to the Virginia Standards of Learning. It is organized around specific topics and includes correlations to the Mathematics Standards of Learning, as well as ideas for assessments and resources. This document is not intended as a script for either curriculum developers or instruction, but it will provide teachers and curriculum developers a place to begin building a curriculum.

This document serves as one of many mathematics resources available to all teachers in the Commonwealth of Virginia, including Algebra Instructional Modules, Geometry Instructional Modules, Connecting Algebra and Biology, Mathematics Standards of Learning for Virginia Public Schools, the Standards of Learning Test Blueprints, the Standards of Learning Test Released Items, and the Mathematics Standards of Learning Curriculum Framework.

#### Overview of the Algebra II Standards of Learning Sample Scope and Sequence

Organizing Topics	Related Standards of Learning
Rational and Radical Equations	AII.1, AII.2, AII.3, AII.7, AII.9, AII.17, AII.20
Systems of Equations and Inequalities	AII.1, AII.11, AII.12, AII.13, AII.14
A Transformation Approach to Relations and Functions	AII.1, AII.4, AII.5, AII.6, AII.8, AII.9, AII.10, AII.15, AII.18, AII.19
Sequences and Series	AII.16

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources	
Rational and Radical	Rational Equations: State the domain and range of a rational function.	AII.9	Individual quizzes Group quizzes	Mathematics SOL Teacher Resource Guide	
Equations	Identify the vertical and horizontal asymptotes of a rational function.	AII.9	Tests Projects Investigations	http://www.pen.k12.va.us/V DOE/Instruction/math_resou rce.html	
	Sketch the graph of a rational function.	AII.9	Portfolios	SOL Test Blueprints	
	Identify a variation as direct, inverse, or joint.	AII.20	Student presentations Questioning strategies	SOL Test Blueprints     SOL Test Released Items	
	Solve practical problems involving joint variation.	AII.20	Peer evaluation	•	Virginia Algebra Resource Center
	Simplify rational algebraic expressions.	AII.7		http://curry.edschool.virginia	
	Compare simplifying rational algebraic expressions to simplifying fractions.	AII.7		.edu/k12/algebra  • NASA	
	Add, subtract, multiply, and divide rational algebraic expressions including complex	AII.7		http://spacelink.nasa.gov/ind ex.html	
	fractions.				The Math Forum <a href="http://forum.swarthmore.edu/">http://forum.swarthmore.edu/</a>
	Solve rational equations algebraically and graphically. The graphing calculator will be used as a primary tool for solution and for	AII.7		• 4teachers http://www.4teachers.org	
	checking the algebraic solution.			Appalachia Educational Laboratory (AEL) <a href="http://www.ael.org/pnp/index">http://www.ael.org/pnp/index</a> <a href="http://www.ael.org/pnp/index">.htm</a>	
				• Eisenhower National Clearinghouse http://www.enc.org/	

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
Rational and Radical Equations (continued)	<ul> <li>Radical Equations: Simplify radical expressions with a variety of indices (rational and integral) until:</li> <li>the index, n, is as small as possible</li> <li>the radicand contains no factors that are the nth powers of an integer or polynomial</li> </ul>	AII.3		
	<ul><li>the radicand contains no fractions</li><li>no radicals appear in the denominator.</li></ul>			
	Rationalize the denominator of a rational expression that contains a radical expression in the denominator.	AII.3		
	Add, subtract, multiply, and divide radical expressions.	AII.3		
	Write expressions in radical form as expressions with rational exponents.	AII.3		
	Evaluate expressions in either exponential or radical form.	AII.3		
	Solve equations containing radical expressions and equations of nth roots algebraically and graphically.	AII.7		
	Check solutions for extraneous roots.	AII.7		
	Recognize a complex number as a number that can be written as $a + bi$ where a and b are real numbers and i is the principal square root of $-1$ .	AII.17		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
Rational and	Recognize pure imaginary numbers.	AII.17		
Radical Equations	Simplify square roots with negative arguments.	AII.17		
(continued)	Represent a complex number geometrically in the coordinate plane.	AII.17		
	Add, subtract, and multiply complex numbers.	AII.17		
	Compare and contrast adding, subtracting, and multiplying complex numbers with operating on real numbers.	AII.17		
	Simplify powers of i and generalize the pattern.	AII.17		
	Simplify rational expressions with complex numbers in the denominator by using complex conjugates.	AII.17		
	Place the following sets of numbers in a hierarchy: complex numbers, pure imaginary numbers, real numbers, rational and irrational numbers, integers, whole numbers and natural numbers. Venn diagrams may be used.	AII.1		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources		
Systems of Equations and	<b>Matrices:</b> Organize data into matrices and identify the dimensions of the matrix.	AII.11	Individual quizzes Group quizzes			Mathematics SOL Teacher Resource Guide
Inequalities	Investigate commutativity and associativity of matrix addition. Compare and contrast matrix addition with addition of real numbers.	AII.1	Tests Projects Investigations Portfolios	http://www.pen.k12.va.us/V DOE/Instruction/math_resou rce.html		
	Multiply matrices using a calculator or a computer with matrix capability.	AII.11	Student presentations Questioning strategies	<ul><li>SOL Test Blueprints</li><li>SOL Test Released Items</li></ul>		
	Solve problems that require matrix multiplication.	AII.11	Peer evaluation	Virginia Algebra Resource Center		
	Investigate commutativity and associativity of matrix multiplication. Compare and contrast	AII.1		http://curry.edschool.virginia .edu/k12/algebra		
	matrix multiplication with multiplication of real numbers.			NASA <a href="http://spacelink.nasa.gov/ind">http://spacelink.nasa.gov/ind</a>		
	Find the determinant of a square matrix.	AII.12		<u>ex.html</u>		
	Identify the identity matrix (I).	AII.12		• The Math Forum http://forum.swarthmore.edu/		
	For a matrix A, find the inverse matrix $A^{-1}$ (if it exists) such that $A * A^{-1} = A^{-1} * A = I$ .	AII.12			4teachers     http://www.4teachers.org	
	Use an inverse matrix to solve matrix equations. Use the graphing calculator or a computer application with matrix capabilities.	AII.12		Appalachia Educational     Laboratory (AEL) <a href="http://www.ael.org/pnp/index">http://www.ael.org/pnp/index</a> <a href="http://www.ael.org/pnp/index">.htm</a>		
				Eisenhower National     Clearinghouse <a href="http://www.enc.org/">http://www.enc.org/</a>		

Virginia Department of Education

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
Systems of Equations and	Compare and contrast solving matrix equations and linear equations.	AII.1		• Algebra Instructional Modules
Inequalities (continued)	Represent a system of equations as a matrix equation where the coefficient matrix times the variable matrix equals the constant matrix.	AII.11	_	<ul><li>Square Patio Patterns</li><li>p. 56</li><li>Matrix Multiplication</li></ul>
	Solve systems of linear equations using inverse matrices. Use the graphing calculator or a computer application with matrix capabilities.	AII.12		<ul> <li>p. 94</li> <li>Matrices (Beginning)</li> <li>p. 96</li> <li>Solving Systems with Matrices p. 98</li> </ul>
	<b>Linear Inequalities:</b> Solve a system of linear inequalities by graphing.	AII.13		<ul><li>Linear Programming</li><li>p. 100</li></ul>
	Identify examples of the properties of inequality and order that occur while solving inequalities.	AII.13		
	<ul> <li>Find the maximum and minimum values of a function over a region (linear programming).</li> <li>Identify the constraints in a practical situation and model them as inequalities.</li> <li>Graph the system of inequalities and identify the area of intersection as the feasible region. The feasible region contains all solutions possible.</li> </ul>	AII.13		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
Systems of Equations and Inequalities (continued)	<ul> <li>(continued)</li> <li>The maximum and minimum values of the function occur at the vertices of the feasible region. Substitute the coordinates of each vertex of the feasible region into the function to determine which vertex yields the maximum (or minimum) value of the function.</li> </ul>			
	Describe the results of a linear programming problem orally and in writing.	AII.13		
	<b>Nonlinear Systems:</b> Solve linear-quadratic systems of equations algebraically and identify the set of ordered pairs that is the solution to the system.	AII.14		
	Solve linear-quadratic systems of equations graphically and identify the set of ordered pairs that is the solution to the system.	AII.14		
	Solve quadratic-quadratic systems of equations algebraically and identify the set of ordered pairs that is the solution to the system.	AII.14		
	Solve quadratic-quadratic systems of equations graphically and identify the set of ordered pairs that is the solution to the system.	AII.14		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
A Transformation Approach to Relations and	Recognize the graphs and equations of parent functions such as $y = x$ , $y = x^2$ , $y = x^3$ , $y =  x $ , $y = a^x$ , step functions, and other polynomial functions.	AII.8/AII.15	Individual quizzes Group quizzes Tests Projects Investigations Portfolios Student presentations Questioning strategies Peer evaluation	Mathematics SOL Teacher Resource Guide http://www.pen.k12.va.us/V DOE/Instruction/math_resou
Functions	Apply transformations (translations, reflections, dilations, and rotations) and combinations of transformations to parent graphs.	AII.8/AII.15		<ul><li>rce.html</li><li>SOL Test Blueprints</li><li>SOL Test Released Items</li></ul>
	Given an image graph of a function, describe the transformations that were performed on the pre-image and the order in which they could have occurred.	AII.8/AII.15		<ul> <li>Virginia Algebra Resource Center <a href="http://curry.edschool.virginia.edu/k12/algebra">http://curry.edschool.virginia.edu/k12/algebra</a></li> </ul>
	Given the equation of a parent graph, vary the coefficients and constants of the equation, observe the changes in the graph of the parent, and generalize the changes to the	AII.8/AII.15		<ul> <li>NASA         http://spacelink.nasa.gov/ind ex.html     </li> <li>The Math Forum</li> </ul>
	graphs of other functions.  Build a strong connection between the algebraic and geometric representations of functions.	AII.8/AII.15		<ul><li><a href="http://forum.swarthmore.edu/">http://forum.swarthmore.edu/</a></li><li><a href="http://www.4teachers.org">4teachers</a></li><li><a href="http://www.4teachers.org">http://www.4teachers.org</a></li></ul>
	Given an equation of a function, sketch the graph of the function.	AII.8/AII.15		<ul> <li>Appalachia Educational Laboratory (AEL) <a href="http://www.ael.org/pnp/index">http://www.ael.org/pnp/index</a></li> </ul>
	Collect data and display it in a scattergraph.	AII.19		<ul> <li>.htm</li> <li>Eisenhower National Clearinghouse http://www.enc.org/</li> </ul>

Virginia Department of Education

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
A Transformation Approach to	Analyze data using measures of central tendency (mean, median, and mode), the range of the data, and box-and-whisker plots.	AII.19		<ul> <li>Algebra Instructional         Modules         Collecting Data and     </li> </ul>
Relations and Functions (continued)	Determine the equation of the curve of best fit using the graphing calculator. Consider the graphs of the parent functions when determining which curve might be appropriate. Use the equation to make predictions.	AII.19		Regression Equations p. 90  Regression Equations p. 92  Investigating Graphs of Polynomials p. 68
	Investigate the commutativity and associativity of combinations of transformations.	AII.1		<ul> <li>Inverse Relationships</li> <li>p. 65</li> <li>Solving Quadratics</li> </ul>
	Given the graph or the equation of a function, identify the domain and range of the function. Include functions with discontinuities.	AII.9/AII.15		Graphically p. 38
	Find the value of a function for a given element in the domain.	AII.9		
	Find the composition of functions algebraically and graphically.	AII.9		
	Find the inverse of a function algebraically and graphically.	AII.9		
	Explain how composition of functions and finding the inverse of a function affects the domain and range of the functions.	AII.9		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
A Transformation	Demonstrate that the exponential and logarithmic functions are inverse functions.	AII.9		
Approach to Relations and Functions (continued)	Factor polynomials completely (difference of two squares, perfect square trinomials, general trinomials, sum and difference of cubes).	AII.5		
	Find the roots of a function algebraically and graphically. Quadratic equations may be solved using a variety of techniques which include but are not limited to factoring to use the zero product property, square roots, the quadratic formula, completing the square, and graphing.	AII.6		
	Investigate the relationship between the solutions of an equation, zeros of a function, x-intercepts, and factors of a polynomial.	AII.10		
	Identify examples of the field properties of real numbers and the properties of equality that occur while solving equations.	AII.1		
	Given the roots of a polynomial, write an equation for the polynomial function.	AII.6		
	Define absolute value.	AII.4		
	Evaluate expressions that contain absolute value.	AII.4		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
A Transformation Approach to	Recognize that the value of the argument for absolute value must be greater than or equal to zero and explain why.	AII.4		
Relations and Functions	Solve absolute value equations algebraically and graphically.	AII.4		
(continued)	Identify examples of the field properties of real numbers and the properties of equality that occur while solving equations.	AII.1		
	Solve absolute value inequalities algebraically and graphically.	AII.4		
	Identify examples of the properties of inequality and order that occur while solving inequalities.	AII.1		
	Conic Sections: Recognize the graphs of the conic sections defined as any figure that can be formed by slicing a double cone (parabola, ellipse, circle, and hyperbola).	AII.18		
	Given an equation, identify the conic section and graph it transformationally.	AII.18		

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
Sequences and Series	Find the next term in a sequence by looking for a pattern.	AII.16	Individual quizzes Group quizzes	Mathematics SOL Teacher Resource Guide
	Find the nth term of an arithmetic sequence and find the position of a given term in an arithmetic sequence.	AII.16	Tests Projects Investigations Portfolios	http://www.pen.k12.va.us/V DOE/Instruction/math_resou rce.html
	Find arithmetic means.	AII.16	Student presentations	SOL Test Blueprints
	Differentiate between a sequence and a series.	AII.16	Questioning strategies Peer evaluation	SOL Test Released Items
	Find the sum of an arithmetic series.	AII.16		Virginia Algebra Resource     Center     http://curry.edschool.virginia
	Find specific terms in an arithmetic series.	AII.16		<u>.edu/k12/algebra</u>
	Use sigma ( $\Sigma$ ) notation to denote sums.	AII.16		NASA <a href="http://spacelink.nasa.gov/ind">http://spacelink.nasa.gov/ind</a>
	Compare and contrast arithmetic and geometric sequences.	AII.16		• The Math Forum
	Find the nth term of a geometric sequence and	AII.16		http://forum.swarthmore.edu/
	the position of a given term in a geometric sequence.			• 4teachers http://www.4te
	Find geometric means.	AII.16		Appalachia Educational
	Find the sum of a geometric series.	AII.16		Laboratory (AEL) <a href="http://www.ael.org/pnp/index">http://www.ael.org/pnp/index</a>
	Find specific terms in a geometric series.	AII.16		<ul> <li>htm</li> <li>Eisenhower National Clearinghouse http://www.enc.org/</li> </ul>

Virginia Department of Education

Organizing Topic	Essential Knowledge and Skills	Related SOL	Sample Classroom Assessment Methods	Sample Resources
Additional Topics in Algebra II	These topics are not assessed on the Algebra II end-of-course assessment, but they may be used to enhance knowledge of some topics in Algebra II and to bring closure to the course.  • Find the sum of an infinite geometric series.			
	Count the number of permutations and combinations possible in a given situation. Use counting techniques in binomial experiments to determine binomial probabilities.			
	Graph points and equations in three dimensions.			
	Solve exponential and logarithmic equations.			
	Use synthetic division and synthetic substitution to find all complex roots of a polynomial function.			
	Study parametric equations and the graphs of parametric equations.			